

NORTHWESTERN UNIVERSITY

Title of the Dissertation

A DISSERTATION

SUBMITTED TO THE GRADUATE SCHOOL  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

for the degree

DOCTOR OF PHILOSOPHY

Field of Mathematics

By

Name of Author

EVANSTON, ILLINOIS

December 2002

© Copyright by Name of Author 2002

All Rights Reserved

# ABSTRACT

Title of the Dissertation

Name of Author

We study the effects of warm water on the local penguin population. The major finding is that it is extremely difficult to induce penguins to drink warm water. The success factor is approximately  $-e^{-i\pi} - 1$ . Replace this text with your own abstract.

## **Acknowledgements**

Text for acknowledgements.

## Preface

This is the preface.

## Contents

|  |      |
|--|------|
| ABSTRACT   | iii  |
| Acknowledgements   | iv   |
| Preface  | v    |
| List of Tables   | vii  |
| List of Figures  | viii |
| Introduction   | 1    |
| Chapter 1. Sample Mathematics and Text                             | 2    |
| 1.1. In-line and Displayed Mathematics                             | 2    |
| 1.2. Mathematics in Section Heads $\int_{\alpha}^{\beta} \ln t dt$ | 3    |
| 1.3. Theorems, Lemmata, and Other Theorem-like Environments        | 3    |
| References   | 6    |
| Appendix . Title of Appendix                                       | 7    |
| 1. First section of Appendix                                       | 7    |
| Appendix. Vita   | 8    |

## List of Tables

|                   |   |
|-------------------|---|
| 1.1 Sample table. | 4 |
|-------------------|---|

## List of Figures

## **Introduction**

This is the introduction. Use a normal chapter heading if you prefer to write the introduction as chapter 1.

## CHAPTER 1

### Sample Mathematics and Text

#### 1.1. In-line and Displayed Mathematics

The expression  $\sum_{i=1}^{\infty} a_i$  is in-line mathematics, while the numbered equation

$$(1.1) \quad \sum_{i=1}^{\infty} a_i$$

is displayed and automatically numbered as equation 1.1.

Let  $H$  be a Hilbert space,  $C$  be a closed bounded convex subset of  $H$ ,  $T$  a nonexpansive self map of  $C$ . Suppose that as  $n \rightarrow \infty$ ,  $a_{n,k} \rightarrow 0$  for each  $k$ , and  $\gamma_n = \sum_{k=0}^{\infty} (a_{n,k+1} - a_{n,k})^+ \rightarrow 0$ . Then for each  $x$  in  $C$ ,  $A_n x = \sum_{k=0}^{\infty} a_{n,k} T^k x$  converges weakly to a fixed point of  $T$ .

Two sets of  $\text{\LaTeX}$  parameters govern mathematical displays.<sup>1</sup> The spacing above and below a display depends on whether the lines above or below are short or long, as shown in the following examples.

A short line above:

$$x^2 + y^2 = z^2$$

and a short line below.

---

<sup>1</sup> $\text{\LaTeX}$  automatically selects the spacing depending on the surrounding line lengths.

A long line above may depend on your margins

$$\sin^2 \theta + \cos^2 \theta = 1$$

as will a long line below. This line is long enough to illustrate the spacing for mathematical displays, regardless of the margins.

### 1.2. Mathematics in Section Heads $\int_{\alpha}^{\beta} \ln t dt$

Mathematics can appear in section heads. Note that mathematics in section heads may cause difficulties in typesetting styles with running headers or table of contents entries.

### 1.3. Theorems, Lemmata, and Other Theorem-like Environments

A number of theorem-like environments is available. The following lemma is a well-known fact on differentiation of asymptotic expansions of analytic functions.

**Lemma 1.** *Let  $f(z)$  be an analytic function in  $\mathbb{C}_+$ . If  $f(z)$  admits the representation*

$$f(z) = a_0 + \frac{a_1}{z} + o\left(\frac{1}{z}\right),$$

*for  $z \rightarrow \infty$  inside a cone  $\Gamma_{\varepsilon} = \{z \in \mathbb{C}_+ : 0 < \varepsilon \leq \arg z \leq \pi - \varepsilon\}$  then*

$$(1.2) \quad a_1 = -\lim_{z \rightarrow \infty, z \in \Gamma_{\varepsilon}} z^2 f'(z).$$

**Proof.** Change  $z$  for  $1/z$ . Then  $\Gamma_\varepsilon \rightarrow \bar{\Gamma}_\varepsilon = \{z \in \mathbb{C}_- : \bar{z} \in \Gamma_\varepsilon\}$  and

$$(1.3) \quad f(1/z) = a_0 + a_1 z + o(z).$$

Fix  $z \in \bar{\Gamma}_\varepsilon$ , and let  $C_r(z) = \{\lambda \in \mathbb{C}_- : |\lambda - z| = r\}$  be a circle with radius  $r = |z| \sin \varepsilon/2$ . It follows from (1.3) that

$$(1.4) \quad \frac{1}{2\pi i} \int_{C_r(z)} \frac{f(\lambda) d\lambda}{(\lambda - z)^2} = \sum_{m=0}^1 a_m \frac{1}{2\pi i} \int_{C_r(z)} \frac{(\lambda - z_0)^m d\lambda}{(\lambda - z)^2} + R(z),$$

where for the remainder  $R(z)$  we have

$$\begin{aligned} |R(z)| &\leq r^{-1} \max_{\lambda \in C_r(z)} o(|z|) = r^{-1} \max_{\lambda \in C_r(z)} |\lambda| \cdot O(|z| + r) \\ &= \frac{|z| + r}{r} \cdot O(|z| + r) = \frac{1 + \sin \varepsilon}{\sin \varepsilon} \cdot O(|z|). \end{aligned}$$

Therefore  $R(z) \rightarrow 0$  as  $z \rightarrow \infty$ ,  $z \in \bar{\Gamma}_{\varepsilon/2}$ , and hence by the Cauchy theorem (1.4) implies

$$\frac{d}{dz} f(1/z) = a_1 + R(z) \rightarrow a_1, \text{ as } z \rightarrow \infty, z \in \bar{\Gamma}_{\varepsilon/2},$$

that implies (1.2) by substituting  $1/z$  back for  $z$ . □

| Head  | Head  | Head  |
|-------|-------|-------|
| entry | entry | entry |
| entry | entry | entry |
| entry | entry | entry |

Table 1.1. Sample table.

Following is a short bibliography. It has no relationship to the previous text, but can be used to show sample citations such as [4] and [6]. This typesetting style places each citation inside square brackets. If you want multiple citations to appear in a single set of square brackets you must type all of the citation keys inside a single citation, separating each with a comma. Here is an example: [2, 3, 4].

## References

- [1] American Petroleum Institute, Technical Data Book - Petroleum Refining, 5th edition, 1992
- [2] Harstad, K. and Bellan, J., "Isolated fluid oxygen drop behavior in fluid hydrogen at rocket chamber pressures", *Int. J. Heat Mass Transfer*, 1998a, **41**, 3537-3550
- [3] Harstad, K. and Bellan, J., "The Lewis number under supercritical conditions", *Int. J. Heat Mass Transfer*, in print
- [4] Hirshfelder, J. O., Curtis, C. F. and Bird, R. B., *Molecular Theory of Gases and Liquids*, John Wiley and Sons, Inc., 1964
- [5] Prausnitz, J., Lichtenthaler, R. and de Azevedo, E., *Molecular thermodynamics for fluid-phase equilibrium*, Prentice -Hall, Inc., 1986
- [6] Reid, R. C., Prausnitz, J. M. and Polling, B. E., *The Properties of Gases and Liquids*, 4th Edition, McGraw-Hill Book Company, 1987

## APPENDIX

### **Title of Appendix**

#### **1. First section of Appendix**

This is the Appendix.

## Vita

This is the Vita (optional).